A NEW LOOK AT THE LANDAU'S THEORY OF SPREADING AND DAMPING OF WAVES IN COLLISIONLESS PLASMAS*

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The theory of plasma waves and Landau damping in Maxwellian plasmas, Landau's "rule of pass around poles" include doubtful statements, particularly related to an artificial "constructing" of the dispersion equation, what should allow the possibility of its solution otherwise not existing at all, and the possibility of analytical continuations of corresponding very specific ruptured functions in the one-dimensional Laplace transformation, used by Landau, what is the base of his theory.

We represent, as an accessible variant, a more general alternative theory based on a two-dimensional Laplace transformation, leading to an asymptotical in time and space solution as a complicated superposition of coupled damping and *non-damping* plane waves and oscillations with different dispersion laws for every constituent mode. This theory naturally and very simply explains paradoxes of the phenomenon of plasma echo. We propose for discussion a new ideology of plasma waves (both electron and ion-acoustic waves) qualitatively different from the traditional theory of Landau damping for non-collisional as well as for low-collisional plasmas.

The sophisticated theory of Landau damping of longitudinal plasma waves, which was published by Landau in 1945 [1], is now considered as the greatest success of the physics of plasma, the heritage verified experimentally and theoretically, which has not to be doubted. But this halo is to a considerable extent dispersed after some critical analysis of different sides of the problem.

1. According to the classical linear theory of longitudinal plasma waves, in order to derive the dispersion equation one has to substitute expressions of the type of $\exp(-i\omega t + ikx)$ for electrical field E and perturbation $f_1(\vec{v}, x, t)$ of the isotropic electron distribution function $f_0(v)$ into the linearized non-collisional kinetic equation

$$\frac{\partial f_1}{\partial t} + v_x \frac{\partial f_1}{\partial x} + \frac{eE_x}{m_e}\frac{\partial f_1}{\partial v_x} = 0, \tag{1}$$

and Poisson's equation

$$\frac{\partial E_x}{\partial x} = 4\pi e \int_{-\infty}^{\infty} f_1 d\vec{v}, \tag{2}$$

where E_x is the selfconsistent electrical field in x direction, v_x is the x-component of the electron velocity, e, m_e are correspondingly the electron charge and mass, ω and k are generally complex; and then to solve the resulting dispersion equation $D(\omega, k) = 0$ defining a dependence $k = f(\omega)$ for electron plasma waves, one or more, according to the number of its roots.

For plasma waves, as it can be immediately shown assuming $\omega = \omega_0 - i\delta$ at real ω_0, δ and k, breaking up integrals according to velocity components in the range $-\infty < \vec{v}_i < \infty$ to constituents $-\infty < \vec{v}_x < 0$ and $0 < \vec{v}_x < \infty$ and picking out the real and imaginary parts, after some very simple procedures: the dispersion equation has no solutions for δ. So, one obtains easily for $D(\omega, k)$ and imaginary part $\operatorname{Im} D(\omega, k) = 0$, determining preferably δ, equations

$$D(\omega, k) = 1 - \frac{4\pi i e^2}{km}\int_{-\infty}^{\infty} \frac{\partial f_0(v)}{\partial v_x}\frac{d\vec{v}}{p + ikv_x} \equiv 1 - iF(p),$$

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$$\operatorname{Im} D(\omega, k) = \frac{16\pi e^2 \omega_0 \delta}{m_e} \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \int_{-\infty}^{\infty} \frac{\partial f_0}{\partial v_x} \frac{v_x dv_x}{(\omega_0^2 - k^2 v_x^2 - \delta^2)^2 + 4\delta^2 \omega_0^2}, \tag{3}$$

where $D(\omega, k)$ is susceptibility, $F(p)$ is a particular case of "Landau function", which will be discussed later, $p = -i\omega$, and $\partial f_0/\partial v_x < 0$. Since near the point $v_x \simeq \omega_0/k$ the integral appears to diverge as $1/\delta$, the whole expression for $\operatorname{Im} D(\omega, k)$ seems to be finite, though ruptured at $\delta \to \pm 0$. But here the integrand in the left-hand side is sign-invariant for both Maxwellian f_0 and any other f_0 with sign-invariatn $\partial f_0/\partial v_x$, so this equation is evidently impossible to satisfy at whatever $\omega_0 \neq 0$ and any δ, including $\delta \to \pm 0$ (in contrary to $\operatorname{Re} D(\omega, k) = 0$). Strongly in $\delta = 0$ this expression makes no sense. This equation has no solutions, too, independently on Landau theory and his analytical continuations of such type of functions, on sophisticated Van Kampen modes and whatever theories else.

The case of a low-collisonal plasma is analogous, but with replacing δ, in the simplest approximation, by $(\delta - \nu)$, where ν is collision frequency, with the same result of non-existing solutions.

This statement is also equivalent to the note in [1] about non-existence of complex poles in the right halfplane of the Laplace transformation used in [1]. In the very strong canonical sense it means indeed non-existence of solutions of the type of a single (or of some number, according to possible roots at fixed k, as in multimode Landau's asymptotical solution) monochromatic plasma waves.

One may also expand the terms of the dispersion equation, after integration over velocity spherical angles $d\varphi d\theta$, in the small parameter $\delta/\omega_0 \ll 1$, but the resulting series is asymptotically divergent (e.g., cf. [2]), and according to the theory of such expansions, the error of a resulting (in this approximation) dispersion equation solution for δ (which just coincides here with the traditional Landau's expression for δ) turns out to be comparable in value with the error, defined by a linear in δ term of the expansion in δ, i.e. the solution error is comparable with this solution itself.

2. Landau's the most slowly damping asymptotical wave should indeed satisfy the dispersion equation (3). But the existence of a solution of the disperation equation is attained by including in it some additional terms defined according to considerations not related to the dispersion equation inself. It is found that such "artificially constructed" (i.e. to the Vlasov integral in the arbitrary sense of principal value one arbitrarily adds the half-residium with semi-circle contour in the complex v_x-plane in the sense of Landau's rule) dispersion equation which just now has solutions, corresponds at asymptotically large times to plane attenuating waves. But it is trivially evident that the dispersion relation does not contain variables x and t, and at $t \to \infty$ the precise dispersion equation does not turn into the modified dispersion equation of Landau theory.

The terms added by Landau [1] are found by a calculation of some contour integral along the real axis in the plane of the complex variable v_x with passing around the pole $-i\delta/k$ near the point ω_0/k of the real axis at $\delta \to +0$ (Fig.1).

a b

Fig. 1 (a) and (b). Some alternative variants of calculation (with different results) of the Landau contour integrals $F(p)$, $p = -i\omega$, in the plane of complex v with $\omega \equiv \omega_0 - i\delta$ (real ω_0, $\delta > 0$) at $\delta \to +0$.

It is easy to see that in dependence on a way of tending δ to zero there are equally accessible, as an example, both possibilities: (a) (half-residium in a pole near the real axis, what is a result of the Landau rule of going around the pole), and (b) (total residium in the pole), as well as other variants, including those leading to possible infinite results. Any discussion about a preferable rightness of either the first or the second result (see e.g. [3]) appears to be objectless because of non-existing any definite limit of the contour integral at $\delta \to 0$ at all. This means, that the result of this calculations depends on a selected way of tending $\delta \to 0$ relative to the contour integral calculation procedure in the complex plane v_x. Because of it an analytical continuation of the regular in the upper halfplane of complex $\omega \equiv ip$ function $F(p)$ into the lower halfplane ω appears to be non existing.

For calculation of expressions of the type (3) at $\delta \to \pm 0$ one might use the relation which is known in mathematics as Sokhotsky formula

$$\lim_{\epsilon \to \pm 0} \int_{-\infty}^{\infty} \frac{f(x)dx}{x + i\epsilon} = \mp i\pi f(0) + \text{v.p.} \int_{-\infty}^{\infty} \frac{f(x)dx}{x} \tag{4}$$

which thus appears to be only a particular case because of the limit existence being related to a selected definite procedure of calculation of an integral (according to the definition) as a limit of the corresponding sum

$$\int_{-\infty}^{\infty} \frac{f(x)dx}{x+i\epsilon} = \lim_{\Delta x_k \to 0} \sum_k \frac{f(x_k)}{x_k + i\epsilon}. \tag{5}$$

Substitution of this definition (5) into the left-hand side of eq. (4) results in a double-limit sum, the value of which, as being a limit, is evidently depending on the arbitrary choice of the relative cooperative rates of tending to zero of the values of Δx_k and ϵ.

Strictly speaking, it takes plase also for the above considered expression (3) $\lim[\mathrm{Im}\, D(w,k)]$ (and $F(p)$) at $\delta \to \pm 0$. So, the above pointed out result of some definite finite limits for that one is only some particular way of calculation of the mentioned double-limit indefinite sum. That is, the expression $\lim[\mathrm{Im}\, D(w,k)]$ in (3) has really no definite sense at all.

3. In the well known paper of Landau [1] the field function $E(x,t)$ was found by solving coupled equations (1) and (2) at an in advance arbitrarily taken coordinate dependence $E(x,t) = \varphi(t).exp(ikx)$ and $f_1 \sim exp(ikx)$ at all the times (cf. also [4],[5]) by means of the one-sided one dimensional Laplace time transformation with the use of an analytical continuation of a function of the form

$$F(p) = \int_{-\infty}^{\infty} \frac{\psi(v)dv}{p+ikv} \equiv F(\omega, k) \tag{6}$$

from the right half-plane of complex p to the left through the peculiar line $\mathrm{Re}\, p \equiv \epsilon = 0$ and with the plane damping wave as an asymptotical solution (v is hereinafter x-component of velocity with omitted subscript x; $\psi(v)$ is a real function of v).

Introducing complex v with the counterclockwise rule of going around the pole $v_p = ip/k$ at $\mathrm{Re}\, p < 0$ (Landau's rule) at integration by v, one removes the rupture at the line $\epsilon \to \pm 0$ defined e.g., as some partial cases, by the Sokhonsky formula (half-residium) or by the residue theorem, but does not remove the rupture of derivatives $dF(p)/dp$ in the analyticity Cauchy-Riemann conditions in the direction which is perpendicular to the line $\mathrm{Re}\, p = 0$. So, at $\mathrm{Re}\, p < 0$ one finds, if, e.g. using the residue theorem in the calculation of the contour integral (6), correspondingly at the left-hand side ($-$) and at the right-hand side ($+$) from the line $\mathrm{Re}\, p = 0$:

$$F_-(p) = \text{v.p.} \int_{-\infty}^{\infty} \frac{\psi(v)dv}{p+ikv} + \frac{2\pi}{k}\psi(ip/k), \qquad (\epsilon \leq 0) \tag{7}$$

(that is the Landau's type analytical continuation to the left), and

$$F_+(p) = \int_{-\infty}^{\infty} \frac{\psi(v)dv}{p+ikv}, \qquad (\epsilon > 0). \tag{8}$$

At the calculation of derivatives across (perpendicularly to) the line $\mathrm{Re}\, p = 0$ with a function ψ having not peculiarities, one finds a finite expression

$$\frac{dF_-(p)}{dp} = \frac{2\pi}{k}\frac{d}{dp}\psi(ip/k), \qquad (\epsilon \to -0), \tag{9}$$

but contrary to it a divergent (infinite) at $\epsilon \to +0$ integral expression (with infinite principal value at $\epsilon = 0$) for the derivative of the analytical at $\epsilon \neq 0$ function (8)

$$\frac{dF_+(p)}{dp} = -\int_{-\infty}^{\infty} \frac{\psi(v)dv}{(p+ikv)^2}, \qquad (\epsilon \to +0). \tag{10}$$

4. To the very hard perceived consequences of the Landau theory one should attribute also the existence of the pole $p = -ikv$, which appears at the Landau procedure of analytical continuation of the Laplace transform of the distribution function $f_1(v,x,t)$. This pole leads to an additive term for the distribution function of the type $H(\vec{v})\exp(ikx - ikvt)$, which one uses for an interpretation of the plasma echo [4]. Dramatic attempts to comprehend and to conciliate this "mysterious" term with the common sense are clearly demonstrated in the textbook [4] and are related to the appearing paradoxicability of the existence of non-damping plasma oscillations in the absence of whatever recovery force, whereas the electric field had to be disappearing due to the Landau damping!

Such f_1, independent on $E(x,t)$, can not satisfy the Poisson equation (2), since its substitution in the right hand side of (2) leads to quite arbitrary functions of time and is not consistent with the field damping in the left hand side. At the same time the paradox of plasma echo appears to be naturally explained in principle in the further developed theory.

5. In analogy with [1] let us consider a semiinfinite (halfspace slab) plasma with an *initial* perturbation

$$f_1(v,x,t) = \begin{cases} g(|\vec{v}|)\exp(ikx) & \text{at } x \geq 0 \\ 0 & \text{at } x < 0, \end{cases} \tag{11}$$

where v is now the velocity component along x, and $g(|\bar{v}|)$ is some integrable function of $|\bar{v}|$.

Equation (1) is easily solved according to the known procedure with the method of characteristics (by reduction to the equivalent system of simple ordinary differential equations) with the result

$$f_1(v,x,t) = g(|v|)e^{ikx} + \int_{(x,x'>0)}^{t} w[x - v(t-t'), t']dt' \tag{12}$$

where

$$w(x,t) \equiv -\frac{eE(x,t)}{m_e}\frac{\partial f_0}{\partial v_x}; \tag{13}$$

$$x' \equiv x - |v|(t-t') \tag{14}$$

and an additional physical condition $x' > 0$ is related to the finite speed of spreading of particles which are arriving to the point x.

Substituting expressions (12) - (14) into eq. (2) one obtains a linear integro-differential equation for $E(x,t)$:

$$\frac{\partial E(x,t)}{\partial x} = \frac{\sqrt{8\pi m_e}e^2 n_e}{k_B T}\int_{-\infty}^{\infty} dvv\exp(-m_e v^2/2k_B T) \times \int_{t_0(v)}^{t} dt' E[x - v(t-t'), t'] - 4\pi e\alpha(x)\exp(ikx) \tag{15}$$

where the dependence $\exp(ikx)$ is supposed only for the sake of correspondence with the assumed by Landau coordinate dependence [1], and $\alpha(x) = \alpha_0 = const$ at $x \geq 0$; $\alpha(x) = 0$ at $x < 0$; $t_0(v) = 0$ at $t - x/|v| < 0$; $t_0(v) = t - x/|v|$ at $t - x/|v| \geq 0$.

The one-value solution of eq. (15) must be defined in the same extent by both initial and boundary conditions. The dependence $E(x,t)$ on t, as well as on x is defined by solving equation (15), and some analogy with the Landau problem [1] for the infinite plasma may be realized by taking the limit $x \to \infty$.

The general solution of equation (15) can be found with onesided two-dimensional Laplace transformation:

$$E(\zeta,\tau) = \frac{1}{(2\pi i)^2}\int_{\sigma_1 - i\infty}^{\sigma_1 + i\infty} e^{p_1\tau}dp_1 \int_{\sigma_2 - i\infty}^{\sigma_2 + i\infty} E_{p_1 p_2} e^{p_2\zeta} dp_2; \tag{16}$$

$$\frac{\partial E(\zeta,\tau)}{\partial\zeta} = \frac{1}{(2\pi i)^2}\int_{\sigma_1 - i\infty}^{\sigma_1 + i\infty} dp_1 \int_{\sigma_2 - i\infty}^{\sigma_2 + i\infty} dp_2 p_2 E_{p_1 p_2} e^{p_1\tau + p_2\zeta} - \frac{E(0,\tau)}{2\pi i}\int_{\sigma_2 - i\infty}^{\sigma_2 + i\infty} \frac{e^{p_2\zeta}}{p_2} dp_2, \tag{17}$$

where $\sigma_1, \sigma_2 > 0$, and there are introduced normalized dimensionless variables

$$\zeta = kx; \qquad \tau = t/t_0; \qquad t_0 = \frac{1}{k}\sqrt{\frac{m_e}{2k_B T}} = \frac{1}{kv_T}; \tag{18}$$

k is real; v_T is thermal mean velocity. Here appears also some $E(0,\tau)$ as a boundary field constant of integration. It can be naturally considered as a given field related to some external circumstances as it is due to some external sources, etc.

After substitution of eqs. (16), (17) and (18) into eq. (15) one obtains

$$\begin{aligned} &-4\pi^2\alpha_1(\zeta)e^{i\zeta} + A\int_{\sigma_1 - i\infty}^{\sigma_1 + i\infty} dp_1 \int_{\sigma_2 - i\infty}^{\sigma_2 + i\infty} dp_2 E_{p_1 p_2} e^{p_1\tau + p_2\zeta} \times \\ &\{\int_0^{\zeta/\tau} d\xi\xi e^{-\xi^2}[\frac{1 - e^{-p_2\xi\tau - p_1\tau}}{p_1 + p_2\xi} - \frac{1 - e^{p_2\xi\tau - p_1\tau}}{p_1 - p_2\xi}] + \int_{\zeta/\tau}^{\infty} d\xi\xi e^{-\xi^2}[\frac{1 - e^{-p_1\zeta/\xi - p_2\zeta}}{p_1 + p_2\xi} - \frac{1 - e^{-p_1\zeta/\xi + p_2\zeta}}{p_1 - p_2\xi}]\} \\ = &\int_{\sigma_1 - i\infty}^{\sigma_1 + i\infty} dp_1 \int_{\sigma_2 - i\infty}^{\sigma_2 + i\infty} dp_2 p_2 E_{p_1 p_2} e^{p_1\tau + p_2\zeta} - 2\pi i E(0,\tau)\int_{\sigma_2 - i\infty}^{\sigma_2 + i\infty} \frac{e^{p_2\zeta}}{p_2} dp_2, \end{aligned} \tag{19}$$

where $A = const$; stepwise function α_1 is proportional to $\alpha(x)$.

Keeping in mind the subsequent consideration of some longitudinal analogy of the skin effect, we shall suppose the field $E(0,\tau)$ takes the form

$$E(0,\tau) = E_0 e^{-i\beta\tau}, \tag{20}$$

where β is some real number.

Taking in account equalities of the type

$$e^{-i\beta\tau} = \frac{1}{2\pi i}\int_{\sigma_1 - i\infty}^{\sigma_1 + i\infty} \frac{e^{ip_1\tau}}{p_1 + i\beta} dp_1, \tag{21}$$

and finiteness of the integrands in points ξ with $p_1 \pm p_2\xi \to 0$ for terms of the type

$$\frac{1 - e^{\pm(p_1 \pm p_2\xi)}}{p_1 \pm p_2\xi},$$

so one assuming the following integrations along ξ in the sense of the principal value, one can find an asymptotical solution of eq. (19) as

$$E_{p_1p_2} = \frac{\frac{\alpha^*}{p_1p_2(p_2-i)} + \frac{E_0}{p_2^2(p_1+i\beta)}}{1 + \frac{2A\bar{\xi}^2 e^{-\bar{\xi}^2}\Delta\xi}{p_1^2 - p_2^2\bar{\xi}^2}} \tag{22}$$

where $\alpha^* \sim \alpha_0$, and $\bar{\xi}$ is some characteristic value of the normalised velocity $\xi \equiv v/v_T$; $\bar{\xi}$ and $\Delta\xi$ are of the order of 1, and

$$A \equiv \frac{4\sqrt{\pi}e^2 n_e}{k^2 k_B T}. \tag{23}$$

In the derivation of eq. (22) it is supposed that one may apply to integrals over ξ in the braces of eq. (19) the mean value theorem, replacing functions of ξ in the integrands with the functions of some constant value $\bar{\xi}$ belonging to the interval of integration. Also at integration over p_1, p_2 one can neglect arising terms

$$\frac{e^{-p_2\bar{\xi}\tau - p_1\tau}}{p_1 + p_2\bar{\xi}}; \qquad \frac{e^{-p_1\zeta/\bar{\xi} - p_2\zeta}}{p_1 + p_2\bar{\xi}}. \tag{24}$$

asymptotically for $\sigma_1, \sigma_2 > 0$ and large $\zeta, \tau \to \infty$. At some fixed value of $\sigma_2 > 0$ Laplace transformation implies the integration contour $\sigma_1 \pm i\infty$ to be at right of the pole $p_1 - p_2\bar{\xi} = 0$ ($\sigma_1 > \sigma_2\bar{\xi}$); this also means the possibility to neglect in an asymptotical solution with large $\zeta, \tau \to \infty$ the exponentially small terms

$$\frac{e^{p_2\bar{\xi}\tau - p_1\tau}}{p_1 - p_2\bar{\xi}}; \qquad \frac{e^{-p_1\zeta/\bar{\xi} + p_2\zeta}}{p_1 - p_2\bar{\xi}}. \tag{25}$$

Asymptotical value of $E(\zeta,\tau)$ at both $\zeta \to \infty$ and $\tau \to \infty$ is then defined by the pair poles of expression (22) at shifting the integration contours σ_1, σ_2 to the left. To the term with $\alpha_0 \neq 0$ there corresponds a constituent of the asymptotical solution $E(\zeta,\tau)$, defined by the residua sum in the poles

$$(p_1 = 0,\ p_2 = i); \qquad (p_1 = 0,\ p_2 = \pm\sqrt{2Ae^{-\bar{\xi}^2}\Delta\xi});$$

$$(p_1 = \pm i\bar{\xi}\sqrt{2Ae^{-\bar{\xi}^2}\Delta\xi + 1},\ p_2 = i) \tag{26}$$

(the poles $(p_1, p_2) = (0, 0)$ do not contribute to $E(\zeta,\tau)$ since $E_{p_1p_2} \to 0$ at successive transits $p_1 \to 0$, then $p_2 \to 0$ or v.v.).

The phase speed of waves, which correspond to the last pair (p_1, p_2) in (26) at small n_e, coincides with the mean thermal velocity

$$\frac{\omega_0}{k} \simeq \frac{1}{t_0 k} = \sqrt{\frac{2k_B T}{m_e}} = v_T,$$

what is in accordance with experimental plasma echo speeds. At large A one finds

$$\frac{\omega_0}{k} \simeq \frac{\sqrt{2A}}{t_0 k} = \frac{\gamma\omega_L}{k} \simeq \frac{\omega_L}{k},$$

where ω_L if Langmuir frequency, and γ is a factor of the order of 1.

For the term in (22) with $E_0 \neq 0$ (with expected extinguishing plane longitudinal waves) one obtains pole pairs defining additional constituents of the asymptotical solution:

$$(p_1 = -i\beta,\ p_2 = 0); \qquad (p_1 = \pm i\bar{\xi}\sqrt{2Ae^{-\bar{\xi}^2}\Delta\xi},\ p_2 = 0);$$

$$(p_1 = -i\beta,\ p_2 = \pm\sqrt{2Ae^{-\bar{\xi}^2}\Delta\xi - (\beta/\bar{\xi})^2}). \tag{27}$$

But in the pole $p_2 = 0$ of the second order (that is $1/p_2^2$) the residium equals zero.

The "mixed" constituent of solution which corresponds to the poles $(p_1 = -i\beta, p_2 = i)$ at $x, t \to \infty$, gives zero contribution.

The derived asymptotical solutions content some exponentially growing modes. This is easy explained by the non-selfconsistency of assumed initial and boundary conditions $f_1(\vec{v}, x, 0)$ and $E(0, t)$ in the absence of an outside source field. The divergent terms can be easily removed with the corresponding fitting amplitude values α^* and E_0 in eq. (22) and assuming $\beta = 0$. So one obtains only nondamping standing oscillations as a sum of modes moving with the Langmuir velocity at large n_e and the one-dimensional mean thermal velocity v_T at small n_e.

One may suppose that an outside source field might be accounted for by dividing the boundary value of the field into a part of the selfconsistent plasma response and the rest one. The first one is defined by condition of the mutual cancellation of terms, which exponentially grow at $x \to \infty$. The second one gives proper plasma modes induced by an outside field.

The above found very simple result of a non-damping standing wave for above variant of the self-consistent initial and boundary conditions seems to be very natural physically.

In a variant of a pulse exitation of longitudinal waves with the field E along x at

$$\alpha(x) = 0; \qquad E(0, \tau) = E_0\delta_+(\tau) \tag{28}$$

one can obtain with the Laplace transformation of δ-function $\delta_+(\tau)$ being 1 (see [6]) a solution

$$E_{p_1p_2} = \frac{E_0/p_2^2}{1 + \frac{2A\bar{\xi}^2 e^{-\bar{\xi}^2}\Delta\xi}{p_1^2 - p_2^2\xi^2}}, \tag{29}$$

respectively only two pair poles

$$(p_1 = i\bar{\xi}\sqrt{2Ae^{-\bar{\xi}^2}\Delta\xi},\ p_2 = 0); \qquad (p_1 = -i\bar{\xi}\sqrt{2Ae^{-\bar{\xi}^2}\Delta\xi},\ p_2 = 0); \tag{30}$$

with zero residua and asymptotic value $E(x, t)$.

These results, which are in principle accessible to experimental verifications, are discrepant qualitatively with the Landau theory, so this means the necessity of a new ideology of plasma waves. The asymptotic limit is represented by a superposition of a finite number of coupled both damping and *non-damping* oscillatory modes with different dispersion laws.

Using the method of Laplace transformation one may easily obtain also asymptotic solution for the perturbation $f_1(\vec{v}, x, 0)$. In this case an arising pole $(p_1 + p_2\bar{\xi})$ and others may indeed lead among others to terms in f_1 of the type $\exp[ik(x - vt)]$, as in the Landau theory, but, in contrary to it, these terms are related to *non-damping* electrical fields.

6. The spreading of transverse electromagnetic waves in a collisionless plasma is traditionally controlled by the Landau rule of passing around the pole and then solving an artificially constructed dispersion equation. But as above, in this case the precise (i.e. without Landau additives) dispersion equation for the linearized and Maxwell field equations, resulting after substitution therein the travelling wave $\exp(-iwt + ikx)$, has no solutions.

The substitution into the Maxwell field equation of the solution of the kinetic equation, which had been obtained with the method of characteristics, leads in non-relativistic approach (after omitting poles terms of the order $(v/c)^2$) to the common optical wave equation for a wave in refractive medium with the refracting index $n = \sqrt{1 - \omega_L^2/\omega_0^2}$, where ω_L is the electron Langmuir frequency, so the solution at real n is represented by the travelling wave

$$E_\perp \sim \exp(-i\omega_0 t + ikx) \tag{31}$$

These results are well known, but it is worth to note however that in the case of medium separating (boundary) surfaces there is possible a peculiar non-orthodox solution of the optical equation, as a differential equation with ruptured boundary conditions, different from the waves (31), but that one is unstable and questionable to be realized experimentally, e.g. in femtosecond spectroscopy [7]. The pole problems do not arise in this consideration at all, whatever singularities in the non-relativistic approach are absent (this corresponds to the well known fact, that there are large v's in poles, so that the Maxwell distribution function $f_0(v)$ there is very small). The pole problem certainly arises in the relativistic case and could be solved by applying the above method of two-dimensional transformation to the relativistic kinetic equation.

As for the experimental verification of the Landau theory, first of all we note that there is a very small number of such works, which also were carried out by the same scientists [8,9]. Such experiments in fact should be very delicate. Certainly, it is very difficult to obtain experimentally a collisionless plasma with Maxwellian electron distribution function due to conflicting demands: the Maxwell distribution is just commonly a consequence of electron energy exchange in collisions, that is the plasma must be collisional. There is also a possibility in principle of a dependence of perturbation spreading in plasma on geometric, spectral and other features of an excitation source.

Conclusions

From an unprejudiced consideration of the Landau theory of spreading and damping of plasma waves there is emphasized fundamental fact that the dispersion equation, derived in accord with the classical canons, does not have whatever solutions, but which *must* exist at least for the most slowly damping Landau's asymptotic travelling wave, be it a case of collisionless or low-collision plasmas. The bypass way, which had been chosen by Landau in 1945 and is now cited in all textbooks on plasma physics almost without variations (cf. [4,5]), respectively Landau's rule of passing around poles, appears to be unsatisfactory because at tending the imaginary part δ of the frequency ω to zero resulting expressions lose their sense since they do not tend to any definite limits. Thus Landau's analytical continuation is really not existent. This is related to the impossibility (e.g. after integration in the dispersion equation by velocity spherical angles $d\varphi d\theta$) of expansion in small δ/ω_0 (that results in asymptotically divergent series, cf. [2]). Landau's theory results in a quite sophisticated and mysterious theory of the plasma echo, which leads to the paradox of conflicting simultaneous existence of a non-damping mode of plasma echo relative to the Landau field-damping wave. At the same time it is possible to construct some reasonable, non-contradictory and mathematically unequivocal theory of spreading of plasma perturbations, using a more strict procedure of the two-dimensional Laplace transformation, which leads asymptotically to a system of coupled damping and non-damping modes of plasma waves and oscillations, but not to an asymptotic single travelling wave with a definite dispersion law, as it should follow from the Landau paradigm.

The rejection of Landau's theory will open many new perspectives in numerous problems of the plasma physics.

One can explain existing now widely adherence to the Landau theory not only by some natural conservatism and piety to Landau, but more by the fact of rather imperceptible, practically unobservable effects arising from this theory in all surrounding us realities (laboratory, technical, experimental, observable cosmic, etc.) in spite of the ultimately wide its diffusion in abstract sophistications of theorists.

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